Aerosonde Hazard Estimation

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We are interested in estimating the probability of an aerosonde hitting a manned aircraft in flight, or an innocent bystander on the ground. We will work through analyses and example calculations for each case.

Symbols

b_s	span of sounding aircraft
b_t	span of transiting aircraft
dA	overlap area
f_c	average crash frequency
f_x	average collision frequency
h	altitude
h_t	target height
l_c	segment length
l_t	target length
P_c	crash probability
P_x	strike probability
R_c	crash radius
T_b	sounding period
V_s	speed of sounding aircraft
V_t	speed of transiting aircraft
y_t	target cross-track position
γ	glide angle
ρ_s	sonde density per unit volume
σ_s	sonde density per unit area
ρ_t	target density per unit volume
σ_t	target density per unit area
σ_y	tracking standard deviation
Φ_s	frontal area of sounding aircraft
Φ_t	frontal area of transiting aircraft
ψ	crossing angle

1 Midair collision

To begin simply, consider a plane in which aerosondes are operating at random. You want to know the probability of collision if you fly across the plane. The calculation involves

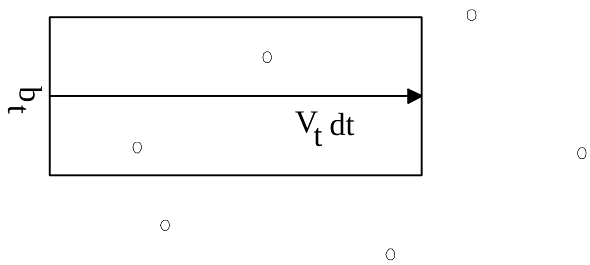

Figure 1: Conflict in 2D with motionless traffic. The aircraft in transit sweeps through the boxed area in time dt. A collision occurs if this area includes one or more targets.

V_t	your speed
b_t	your wingspan
V_s	the aerosondes' speed
b_s	the aerosondes' wingspan
σ_s	the average density of aerosondes over the plane

To simplify further, temporarily restrict attention to the most elementary case in which V_s and b_s are zero. Then figure 1 shows how to compute the collision risk directly. In time dt you sweep through area

$$dA = b_t V_t dt \tag{1}$$

The probability of collision during the interval is

$$\begin{aligned} P_x(dt) &= 1 - e^{-\sigma_s b_t V_t dt} \\ &\approx \sigma_s b_t V_t dt, \qquad \sigma_s b_t V_t dt << 1 \end{aligned} \tag{2}$$

Hence the probability of collision per unit time is

$$f_x = \frac{dP_x}{dt} = \sigma_s b_t V_t \tag{3}$$

As illustrative numbers, take

V_t	860 km/h (typical airliner cruise speed)
b_t	0.06 km (747)
σ_s	$8 \times 10^{-6}/\mathrm{km}^2$ (US radiosonde station density)

f_x then works out to $4 \times 10^{-}4$/flight-hour, or once every 2500 hours on average. Roughly speaking, this is the collision rate that we would have if there were a tethered balloon flying permanently over each radiosonde site (and if no avoiding action were taken!). Fortunately, balloons are not tethered. This reduces the collision probability substantially, as we will demonstrate presently.

First, however, we consider the case in which aerosondes have nonzero speed. This turns out not to make much difference if V_s is small compared with V_t. The effect, as illustrated in figure 2, is to change the overlap area dA as a function of the angle between the two velocity vectors. In particular

$$dA(\psi) \approx b_t \sqrt{V_t^2 + V_s^2 + 2V_tV_s \cos\psi} \;\; dt \tag{4}$$

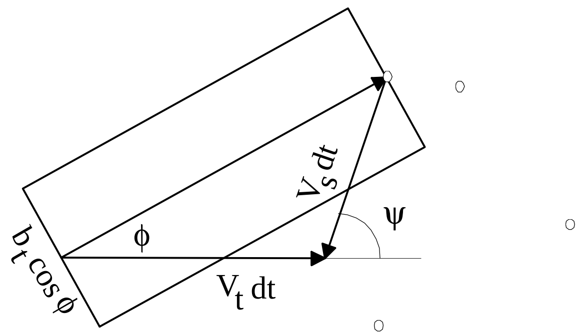

Figure 2: Conflict in 2D with moving traffic. The combined velocity vectors vary the area overlapped in time dt as a function of angle ψ.

(We neglect the complicating factor of $\cos\phi$ from figure 2, so this expression is slightly conservative.) The traffic moves in random directions, so all values of ψ are equally likely. Hence averaging over all possible directions gives

$$\begin{aligned} d\bar{A} &\approx \frac{1}{2\pi}\int_0^{2\pi} dA(\psi)\; d\psi \\ &\approx b_t V_t dt \left(\frac{1}{2\pi}\int_0^{2\pi} \sqrt{1+\left(\frac{V_s}{V_t}\right)^2 + 2\frac{V_s}{V_t}\cos\psi}\;\; d\psi \right) \end{aligned} \tag{5}$$

If V_s/V_t increases from zero to 0.5, for example, the bracketed coefficient increases from 1 to about 1.3. For order-of-magnitude estimation this change is negligible, so we are justified in using the simpler formula in (3).

The effect of finite aerosonde wingspan is also not significant if it is small compared with b_t. But in any case it is easy to include, since one need only replace b_t in (3) with $(b_t + b_s)$.

1.1 3D collision probability

The real 3D problem differs only slightly from the 2D idealisation. Aircraft fly at shallow angles, so the principal component of converging velocity will remain horizontal. The correction required for 3D is only to replace areal density by volumetric density, and target width by frontal area. Thus (3) becomes

$$f_x = \rho_s(\Phi_t + \Phi_s)V_t \tag{6}$$

Returning to the radiosonde example, suppose that each station releases balloons at intervals T_b, and that they rise through the altitudes of interest at vertical speed $\dot{h}$. Then their density per unit length of vertical column, on average, is $1/(T_b\dot{h})$. Hence

$$\rho_s = \frac{\sigma_s}{T_b\dot{h}} \tag{7}$$

Approximate numbers are

T_b	12 hours
$\dot{h}$	20 km/h
Φ_t	$2\times10^{-4}\text{km}^2$ (747, conservatively)

Table 1: Density limits on randomly-distributed aerosondes

	airliners	general aviation
Φ_t [km^2]	2×10^{-4}	10^{-5}
V_t [km/h]	860	200
f_x [per flight-hr]	10^{-9}	10^{-7}
ρ_s [per km^3]	10^{-8}	10^{-4}

Then f_x is 6×10^{-9}/flight-hour. Again this presumes a random distribution of sondes, and no possibility of avoiding action.

Notice, incidentally, that while we have been doing this calculation from the standpoint of the transiting aircraft, the thinking would have been exactly the same had we taken a sonde's point of view. We would simply make the densities appearing in (3) and (6) those of transiting aircraft rather than sondes. Hence the collision risk per sonde flight-hour would be the same as the collision risk per transiting-aircraft flight-hour only if both types were distributed with the same average density.

We can solve (6) for the density which will lead to a specified collision frequency; it is

$$\rho_s = \frac{f_x}{(\Phi_t + \Phi_s)V_t} \tag{8}$$

Results for two cases are listed in table 1. The first uses 747 parameters and sets f_x at 10^{-9}/flight-hour, this being the maximum rate of catastrophic failure considered acceptable by the US FAA. The second uses general-aviation aircraft parameters and sets f_x at 10^{-7}/flight-hour, which is closer to the historical rate achieved by the "see-and-avoid" paradigm. The allowable sonde density calculated in the second case is much higher than any we would expect in practice, so the probability of collision would be negligible by current standards even if nothing were done about avioidance. In the more conservative case, however, the calculated density is marginally lower than would be desired for routine soundings. Hence it would be necessary to arrange for the sonde distribution to be rarefied in areas with transiting aircraft (*e.g.* oceanic tracks). This can easily be arranged, since airways and other busy airspace are well defined over the regions of interest. Aerosondes can be programmed to avoid them laterally or vertically. Note that this entails no requirement for the avoidance strategy to be perfectly reliable. Even if it worked only 90% of the time, it would reduce the aerosonde density tenfold in the areas of concern.

2 Crash hazard

Aircraft crash from time to time and consequently constitute a hazard to innocent bystanders on the surface. Suppose that the average frequency of crashes is f_c. Then the probability of a crash in any interval dt is

$$\begin{aligned} P_c(dt) &= 1 - e^{-f_c dt} \\ &\approx f_c dt, \qquad f_c dt << 1 \end{aligned} \tag{9}$$

Thus the probability of crashing in the interval required to cross a target of length l_t would be

$$P_c(l_t) = f_c \frac{l_t}{V_s} \tag{10}$$

Meanwhile the probability of such a target actually being in the flight path during this interval is the product of

$(b_s + b_t)$	combined width of aircraft and target;
$V_s dt$	length along the track; and
σ_t	average density of targets on the surface.

c.f. (3) and figure 1. Thus the overall probability of a strike, per unit of flight time, is

$$\begin{aligned} f_x &= \left(f_c \frac{l_t}{V_s} \right) (b_s + b_t) V_s \sigma_t \\ &= f_c l_t (b_s + b_t) \sigma_t \end{aligned} \tag{11}$$

For illustration, consider the hazard to ships arising from aerosondes doing reconnaissance over the high seas. Rough numbers are

σ_t	$4 \times 10^{-4}/\text{km}^2$ (10^5 ships, randomly distributed over the oceans)
l_t	0.1 km, averaged over all ship sizes and orientations
b_t	$= l_t$, all orientations being equally likely
b_s	$<< b_t$
f_c	10^{-3}/flight-hour (mostly due to severe weather)

Then f_x is about 4×10^{-9}/flight-hour. Meteorological requirements ultimately may entail about 10^6 annual aerosonde hours in oceanic reconnaissance; at this rate ships would be hit on average once every 250 years. Actually as a hazard estimate this is pessimistic: the probability of seriously *damaging* a ship, as opposed to simply hitting it, would be a good deal smaller. But 10^{-9} is already small enough to be negligible.

Note that the hazard probability in this case is very much less than the aircraft crash rate – a situation obviously different from that in manned aircraft! The hazard probability becomes comparable with the crash rate only if the target density is high. Thus, as the ship example illustrates, reliability requirements can be substantially relaxed if operations are planned to avoid high-density areas.

As a further example, consider a flight-plan leg designed to keep an aircraft over reasonably sparse terrain. There will be some error in tracking the leg, and the aircraft may overfly a few bystanders. We can calculate the associated hazard as follows. Take the tracking error to be Gaussian with standard deviation σ_y. The probability of crossing a bystander of width b_t at a distance y_t from the track centreline is

$$p(y_t) = \frac{1}{\sigma_y \sqrt{2\pi}} \int_{y_t-(b_t+b_s)/2}^{y_t+(b_t+b_s)/2} e^{-1/2(y/\sigma_y)^2}\, dy \tag{12}$$

$$\approx \frac{b_t + b_s}{\sigma_y \sqrt{2\pi}}\, e^{-1/2(y_t/\sigma_y)^2}, \qquad b_t + b_s << \sigma_y \tag{13}$$

Following (11), the probability of a strike is then

$$P_x \approx f_c \frac{l_t}{V_s} \frac{b_t + b_s}{\sigma_y \sqrt{2\pi}}\, e^{-1/2(y_t/\sigma_y)^2} \tag{14}$$

To complete the example with some variety, suppose that bystanders are high rather than long, and so much more likely to be hit from the side than from above. Then

$$l_t \approx \frac{h_t}{\gamma} \tag{15}$$

γ being the flight-path angle. Now take

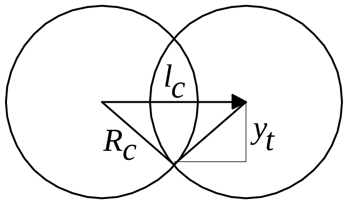

Figure 3: Following failure causing loss of guidance, a crash could occur anywhere within gliding range of the track. A bystander offset by y_t from the original track is at risk if such failures occur on a segment of length l_c.

V_s	80 km/h (typical aerosonde)
σ_y	0.05 km (consistent with flight test experience)
γ	1/20 (at best L/D, hence conservative for most failures)
b_t	0.03 km (typical house)
h_t	0.006 km (typical house)
f_c	10^{-3}/flight-hour
y_t	$3\sigma_y$

With these numbers the probability of a strike turns out to be about 4×10^{-9}. On average, one of every 200 million such bystanders passed at this range would be hit.

This result might be questioned on the basis that some failures (*e.g.* flight computer) would cause loss of tracking performance. In that case a deadman's switch would kill the engine, but the aircraft could crash (with equal probability) anywhere within gliding range. Hence the crash radius is

$$R_c = \frac{h}{\gamma} \tag{16}$$

Figure 3 shows the situation from the point of view of the innocent bystander. He is at risk if the failure occurs anywhere on a segment of length

$$\begin{aligned} l_c &= 2\sqrt{R_c^2 - y_t^2} \\ &= 2R_c\sqrt{1 - (y_t/R_c)^2} \end{aligned} \tag{17}$$

The probability of a failure on this segment is given by (10). If the failure occurs, then the bystander's probability of being struck is just his fraction of the affected area, *i.e.*

$$\sigma_t = \frac{(l_t + b_s)(b_t + b_s)}{\pi R_c^2 + 2R_c l_c} \tag{18}$$

Hence the overall probability of striking the bystander is

$$P_x(y_t) = 2f_c \, \frac{(l_t + b_s)(b_t + b_s)}{V_s R_c} \, \frac{\sqrt{1 - (y_t/R_c)^2}}{\pi + 4\sqrt{1 - (y_t/R_c)^2}} \tag{19}$$

Now suppose that bystanders are randomly distributed across the track, with average areal density σ_t. The average strike probability for all bystanders is

$$\begin{aligned}\bar{P_x} &= 2f_c\,\frac{(l_t+b_s)(b_t+b_s)}{V_sR_c}\left(\frac{1}{2}\int_{-1}^{1}\frac{\sqrt{1-\bar{y}^2}}{\pi+4\sqrt{1-\bar{y}^2}}\,d\bar{y}\right)\\ &= 0.24f_c\,\frac{(b_t+b_s)}{V_sR_c} \qquad (20)\end{aligned}$$

The average number of bystanders at risk per unit time is

$$\frac{dN}{dt}=2V_sR_c\sigma_t \qquad (21)$$

Hence the average rate of bystander strikes is

$$f_x=0.48f_c(l_t+b_s)(b_s+b_t)\sigma_t \qquad (22)$$

This is essentially the same result as we had earlier (11). Note that altitude doesn't appear, except indirectly in the sense that the higher the altitude, the wider the corridor (16) over which the bystander density must be calculated.

Suppose that we want to keep the average strike rate below 10^{-7}/flight-hour, which seems a reasonable guess at the present rate for manned aircraft. What restriction must be imposed on the areas overflown? We use the same numbers as in our last example, except with a failure rate of 10^{-4}/flight-hour, rather than 10^{-3}, because we are accounting only for events that cause uncontrolled departure from track. (The factor of ten is a minimum requirement dictated by economics. In estimating the costs of meteorological reconnaissance by aerosonde, we presume that most attrition will be caused by adverse weather. If systems failures were to cause attrition at a comparable level, then the economics could be improved by making the design more reliable. Hence for minimum cost the systems-failure rate must be made small compared to the overall loss rate, *i.e.* no worse than 10^{-4}/flight-hour.)

At this rate the maximum allowable σ_t turns out to be about 1 house/km^2. Obviously this means that overland operations must be conducted in rural or remote surroundings – but then economical access to such areas is the whole purpose of the aerosonde project.